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Three-Family $SU(5)$ Grand Unification in String Theory

Zurab Kakushadze[1*] and S.-H. Henry Tye[2†]
[1] *Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138*
and
Department of Physics, Northeastern University, Boston, MA 02115
[2] *Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853-5001*

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Abstract

We present two 3-family $SU(5)$ grand unified models in the heterotic string theory. One model has 3 chiral families and 9 pairs of $\mathbf{5}+\overline{\mathbf{5}}$ Higgs fields, and an asymptotically-free $SU(2)\otimes SU(2)$ hidden sector, where the two $SU(2)$s have different matter contents. The other model has 6 left-handed and 3 right-handed $\mathbf{10}$s, 12 left-handed and 9 right-handed $\overline{\mathbf{5}}$s, and an asymptotically-free $SU(3)$ hidden sector. At the string scale, the gauge couplings g^2 of the hidden sectors are three times as big as that of $SU(5)$. In addition, both models have an anomalous $U(1)$.

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*E-mail: zurab@string.harvard.edu

†E-mail: tye@hepth.cornell.edu

If the superstring theory is relevant to nature, it must contain the standard model of strong and electroweak interactions as part of its low energy effective field theory (by low energy, we mean below the string scale). There are at least three possibilities [1]. The string model contains:

- (i) the standard $SU(3) \otimes SU(2) \otimes U(1)$ model;
- (ii) a so-called "guided" grand unified model, *e.g.*, flipped $SU(5)$ [2]; or
- (iii) a grand unified theory (GUT).

It is natural to incorporate supersymmetry into the above cases. To have dynamical supersymmetry breaking, we also need a hidden sector that is strongly interacting at some scale above the electroweak scale. In the four-dimensional heterotic string theory, the total rank of the gauge symmetry (with $U(1)$ counted as 1) must be less than or equal to 22. In the first two possibilities [3], typically there are large hidden sectors. It then follows that there are numerous choices of the hidden sector, and detailed dynamical analyses are needed to distinguish one from another.

In the third case, *i.e.*, grand unified string theory (GUST), the situation is somewhat different. To incorporate both chiral fermions and adjoint (or higher representation) Higgs fields (needed to break the grand unified gauge group to the standard model in the effective field theory), the grand unified gauge symmetry must be realized with a higher-level current algebra. In the four-dimensional heterotic string, the room for gauge symmetry is limited. Since a higher-level current algebra takes up extra room, there is less room for the hidden sector, and hence fewer possiblities. Level-2 string models have been extensively explored in the literature [4]. So far, all the known level-2 GUSTs either have an even number of chiral families, or additional exotic chiral matter some of which would remain light after the electroweak breaking. This property simply follows from the fact that a $\mathbf{Z}_2$ orbifold [5] needed to reach a level-2 current algebra has an even number of fixed points, and this number is closely related to the number of chiral families. Recently, 3-family grand unified string models were constructed [6,7], using level-3 current algebras. The number of possiblities is very limited. In Ref [7], we give one E_6 model and three $SO(10)$ models. They all have $SU(2)$ as the hidden sector gauge group. This $SU(2)$ is asymptotically-free, so that there is a chance of dynamical supersymmetry breaking via gaugino condensation [8]. Because the gauge coupling g^2 of this $SU(2)$ is three times bigger than that of the grand unified gauge group at the string scale, the hidden sector does seem to get strong at a scale above the electroweak scale. Of course, the viability of these models requires more careful analyses, in the framework of string phenomenology [9].

A simple robust way to stabilize the dilaton expectation value is to have a semi-simple hidden sector, with more than one gaugino condensate [10]. So it is interesting to ask if one can get a larger hidden sector in the 3-family GUST construction, in particular, a hidden sector with more than one gaugino condensate. In a separate paper, we shall give a classification of all 3-family $SO(10)$ and E_6 models obtainable from our approach. Since a level-3 current algebra takes up more room than the corresponding level-2 current algebra, the room for the hidden sector in the 3-family GUSTs is severely limited. In our classification, the biggest (and only) asymptotically-free hidden sector gauge symmetry is $SU(2)$. (There is an $SO(10)$ model with $SU(2)^3$ as its hidden sector. None of these $SU(2)$s are asymptotically-free at the string scale. However, it is possible that some of their matter fields become massive via spontaneous symmetry breaking, so the $SU(2)$s can

become asymptotically-free below that mass scale.) So an obvious possibility of having a hidden sector with more than one asymptotically-free gauge group in a 3-family GUST is to consider $SU(5)$ grand unification [11]. Since the $SU(5)$ gauge group takes up less room in the heterotic string, more room may be freed up for the hidden sector. Indeed, one can get 3-family $SU(5)$ GUSTs with larger hidden sectors. In this paper, we shall present such a model, with a $SU(2) \otimes SU(2)$ hidden sector, where both $SU(2)$s are asymptotically-free, but they have different matter contents. The construction of this model automatically yields another closely related 3-family $SU(5)$ model with $SU(3)$ as its hidden sector.

All the $SU(5)$ models in Ref [7] can be obtained from the spontaneous symmetry breakings of the $SO(10)$ models, so the hidden sector remains $SU(2)$. To obtain a larger hidden sector in a 3-family $SU(5)$ GUST, we shall turn on a Wilson line in the 3-family $SO(10)$ GUST first presented in Ref [6]; this Wilson line enhances the gauge symmetry in the hidden sector and at the same time breaks the $SO(10)$ down to $SU(5) \otimes U(1)$. Depending on the details of the construction, there appear two 3-family $SU(5)$ models, with gauge symmetries $SU(3)_1 \otimes SU(5)_3 \otimes U(1)^4$ (the $F1(1)$ model) and $SU(2)_1 \otimes SU(2)_1 \otimes SU(5)_3 \otimes U(1)^4$ (the $F2(1)$ model). Their massless spectra are given in Table I. Since their properties are closely related to the 3-family $SO(10)$ GUST (the $T1(1)$ model), its massless spectrum is also reproduced in Table I to facilitate comparison. All the $U(1)$ charges are normalized so that the lowest allowed value is ± 1, with conformal highest weight $r^2/2$. The radius r for each $U(1)$ is given at the bottom of Table I.

Besides enhanced hidden sectors, there are two new features that arise in these two $SU(5)$ models, in comparison to other 3-family GUSTs. The first new feature is the way the net 3 chiral families appear. The structures of chiral families of these two $SU(5)$ are quite different from that of the E_6, $SO(10)$, $SU(5)$ and $SU(6)$ models of Refs [6,7]. The second new feature is the presence of an anomalous $U(1)$. In string theory, this $U(1)$ anomaly is well understood via the Green-Schwarz mechanism [12]. In contrast, all the known 3-family E_6 and $SO(10)$ models are completely anomaly-free. One of the features that these new models have in common with the $SO(10)$ and E_6 models is that there is only one adjoint in the grand unified gauge group. Also note that in all of these models the adjoint carries no other quantum numbers [13].

The $SU(5)$ model given in the third column of Table I (the $F2(1)$ model) has 3 left-handed and no right-handed $\mathbf{10}$s, and 12 left-handed and 9 right-handed $\overline{\mathbf{5}}$s of $SU(5)$. This means the $F2(1)$ model has 3 chiral families and 9 pairs of Higgs fields in the fundamental representation. One of the $U(1)$s, namely, the third one, is anomalous in this model. The total anomaly is given by $(0, 0, +72, 0)_L$ (with normalization radius $r = \frac{1}{3\sqrt{2}}$). The first two $U(1)$s play the role of the messenger sector, while the last $U(1)$ is part of the visible sector. From Table I, we see that the two $SU(2)$s in the hidden sector have different matter contents, so they are expected to have different threshold corrections. Naively, the first $SU(2)$ in the hidden sector gets strong at a scale a little above the electroweak scale, while the second $SU(2)$ is still weak at this scale. However, this is not expected to happen. Instead, the anomalous $U(1)$ gauge symmetry is expected to be broken by the Fayet-Iliopoulos term [14] at some scale (say, slightly below the string scale). Presummably a number of scalar fields will develop vacuum expectation values without breaking supersymmetry. In particular, there is only one (non-abelian) singlet (namely the $(\mathbf{1}, \mathbf{1}, \mathbf{1})(0, 0, -6, 0)_L$) in the massless spectrum of this model that has an anomalous $U(1)$ charge only. Its scalar component presumably

acquires a vacuum expectation value at this energy scale for the Higgs mechanism. As a result, a number of the $SU(2)$ doublets (but not all) will pick up comparable masses. Below this mass scale, the two $SU(2)$s now have larger β coefficients, so both will become strong above the electroweak scale. However, they will become strong at different scales, each with its own gaugino condensate [9]. A more careful analysis is clearly needed to see if supersymmetry breaking happens in a way that satibilizes the dilaton expectation value to a reasonable value [10].

It remains an open question if there are other 3-family GUSTs that have hidden sectors with multi-gaugino condensates. A classification of all 3-family $SU(5)$ models will be very useful. $SU(6)$ GUST is another possibility. In any case, it is clear that the total number of such models will be very limited.

The $SU(5)$ model given in the second column of Table I (the $F1(1)$ model) has 6 left-handed and 3 right-handed $\mathbf{10}$s, and 12 left-handed and 9 right-handed $\overline{\mathbf{5}}$s of $SU(5)$. The first three $U(1)$s of the $F1(1)$ model are anomalous, whereas the last $U(1)$, which is part of the visible sector, is anomaly-free. The total $U(1)$ anomaly is given by $(+36,-108,-36,0)_L$ (with their normalization radii given in Table I). One can always rotate the charges so that only one $U(1)$ is anomalous. However, there is no singlet that is charged only under this anomalous $U(1)$. As a result, we expect that all three $U(1)$s will be broken. Since the two anomaly-free $U(1)$s (combinations of the first three $U(1)$s) play the role of the messenger sector, the messenger sector scale in this model may be quite high (of the order of string or GUT scale). This is not necessarily a problem since the $SU(3)$ hidden sector will then become strong also at a rather high scale.

Since the construction of the $SU(5)$ GUSTs here is based on the $SO(10)$ GUSTs given in Refs [6,7], and the approach is very similar, the discussion presented below shall be self-contained but relatively brief. The construction uses the asymmetric orbifold framework [5]. First we shall review the construction of the $SO(10)$ model. Our starting point is a $N=4$ space-time supersymmetric Narain model [15], which we will refer to as $N0$, with the lattice $\Gamma^{6,22}=\Gamma^{2,2}\otimes\Gamma^{4,4}\otimes\Gamma^{16}$. Here $\Gamma^{2,2}=\{(p_R||p_L)\}$ is an even self-dual Lorentzian lattice with $p_R,p_L\in\tilde{\Gamma}^2$ ($SU(3)$ weight lattice), and $p_L-p_R\in\Gamma^2$ ($SU(3)$ root lattice). Similarly, $\Gamma^{4,4}=\{(P_R||P_L)\}$ is an even self-dual Lorentzian lattice with $P_R,P_L\in\tilde{\Gamma}^4$ ($SO(8)$ weight lattice), $P_L-P_R\in\Gamma^4$ ($SO(8)$ root lattice). Γ^{16} is the Spin(32)/$\mathbf{Z}_2$ lattice. This model has $SU(3)\otimes SO(8)\otimes SO(32)$ gauge group.

Next we turn on Wilson lines that break the $SO(32)$ subgroup to $SO(10)^3\otimes SO(2)$:

$$U_1=(\vec{e}_1/2||\vec{0})(\mathbf{s}'||\mathbf{0}')(\mathbf{s}|\mathbf{0}|\mathbf{0}|C)\ , \tag{1}$$

$$U_2=(\vec{e}_2/2||\vec{0})(\mathbf{c}'||\mathbf{0}')(\mathbf{0}|\mathbf{s}|\mathbf{0}|C)\ . \tag{2}$$

Here we are writing the Wilson lines as shift vectors in the $\Gamma^{6,22}$ lattice. Thus, both U_1 and U_2 are order two ($\mathbf{Z}_2$) shifts. The $\Gamma^{2,2}$ right-moving shifts are given by $\vec{e}_1/2$ and $\vec{e}_2/2$ ($\vec{e}_1$ and $\vec{e}_2$ being the simple roots of $SU(3)$, while $\vec{0}$ is the identity (null) weight); the left-moving $\Gamma^{2,2}$ momenta are not shifted. The $\Gamma^{4,4}$ right-moving shifts are given by $\mathbf{s}'$ and $\mathbf{c}'$ ($\mathbf{0}'$, $\mathbf{v}'$, $\mathbf{s}'$ and $\mathbf{c}'$ are the identity, vector, spinor and conjugate spinor weights of $SO(8)$, respectively); the left-moving $\Gamma^{4,4}$ momenta are not shifted. The $SO(32)$ shifts are given in the $SO(10)^3\otimes SO(2)$ basis ($\mathbf{0}(0)$, $\mathbf{v}(V)$, $\mathbf{s}(S)$ and $\mathbf{c}(C)$ are identity, vector, spinor and anti-spinor weights of $SO(10)(SO(2))$, respectively). These Wilson lines break the gauge

symmetry down to $SU(3)\otimes SO(8)\otimes SO(10)^3\otimes SO(2)$ in the resulting $N=4$ Narain model, which was referred to as $N1$. All the gauge bosons come from the unshifted sector, whereas the shifted sectors give rise to massive states only.

Before orbifolding the $N1$ model, we will specify the basis that will be used in the following. The right-moving $\Gamma^{2,2}$ momenta will be represented in the $SU(3)$ basis. The twist corresponding to $2\pi/3$ rotations of these momenta will be denoted by θ. We will use the $SU(3)\supset SU(2)\otimes U(1)$ basis for the left-moving momenta corresponding to the $\Gamma^{2,2}$ sublattice. In this basis $\mathbf{1}=\mathbf{1}(0)+\mathbf{2}(3)+\mathbf{2}(-3)$, $\mathbf{3}(\overline{\mathbf{3}})=\mathbf{1}(\mp 2)+\mathbf{2}(\pm 1)$. Here the ireps of $SU(3)$ (identity $\mathbf{1}$, triplet $\mathbf{3}$ and anti-triplet $\overline{\mathbf{3}}$) are expressed in terms of the irreps of $SU(2)$ (identity $\mathbf{1}$ and doublet $\mathbf{2}$) and the $U(1)$ charges are given in brackets. They are normalized to the radius $r=1/\sqrt{6}$; so that the conformal dimension of a state with $U(1)$ charge Q is $h=(rQ)^2/2$.

The right-moving $\Gamma^{4,4}$ momenta will be represented in the $SO(8)\supset SU(3)\otimes U(1)^2$ basis. In this basis, the $SO(8)$ momenta have two $\mathbf{Z_3}$ symmetries. The first $\mathbf{Z}_3$ symmetry is that of $2\pi/3$ rotations in the $SU(3)$ subgroup of $SO(8)$. The second $\mathbf{Z}_3$ symmetry corresponds to a $2\pi/3$ rotation in the $U(1)^2$ plane. Under this rotation $\mathbf{v}'\to\mathbf{s}'\to\mathbf{c}'\to\mathbf{v}'$, which is the well-known triality of the $SO(8)$ Dynkin diagram. In the following, the twist corresponding to the simultaneous $2\pi/3$ rotations in both $SU(3)$ and $U(1)^2$ subgroups will be denoted by Θ.

The right-moving $\Gamma^{4,4}$ momenta will be represented in the $SO(8)\supset SU(2)^4$ basis. In this basis, under cyclic permutations of, say, the first three $SU(2)$s, the weights $\mathbf{v}'$, $\mathbf{s}'$ and $\mathbf{c}'$ are permuted. This is the same as one of the $\mathbf{Z}_3$ symmetries of $SO(8)$ that we considered in the $SU(3)\otimes U(1)^2$ basis, namely, the triality symmetry of the $SO(8)$ Dynkin diagram. In the following, we will denote this outer automorphism of the first three $SU(2)$s by $\mathcal{P}_2$. We can conveniently describe the $\mathcal{P}_2$ outer automorphism as a $\mathbf{Z}_3$ twist. Let η_1, η_2, η_3 and η_4 be the real bosons of $SU(2)^4$. Next, let $\Sigma=(\eta_1+\omega^2\eta_2+\omega\eta_3)/\sqrt{3}$, so its complex conjugate $\Sigma^\dagger=(\eta_1+\omega\eta_2+\omega^2\eta_3)/\sqrt{3}$ (here $\omega=\exp(2\pi i/3)$), and $\rho=(\eta_1+\eta_2+\eta_3)/\sqrt{3}$. In this basis, we have one complex boson Σ, and two real bosons ρ and η_4. The outer automorphism of the first three $SU(2)$s that permutes them is now a $\mathbf{Z}_3$ twist of $\Sigma(\Sigma^\dagger)$ that are eigenvectors with eigenvalues ω (ω^2). Meantime, ρ and η_4 are invariant under this twist, and therefore unaffected. We also note that the $SU(2)$ momenta can be expressed in terms of a one dimensional lattice $\{n/\sqrt{2}\}$, where odd values of n correspond to the states in the $\mathbf{2}$ irrep, whereas the even values describe the states in the idenity $\mathbf{1}$ of $SU(2)$.

Finally, we turn to the $SO(10)^3$ subgroup. To obtain $SO(10)_3$ from $SO(10)^3$, we must mod out by their outer automorphism. In the following we will denote this outer automorphism by $\mathcal{P}_{10}$. Let the real bosons ϕ^I_p, $I=1,...,5$, correspond to the $p^{\rm th}$ $SO(10)$ subgroup, $p=1,2,3$. Then $\mathcal{P}_{10}$ cyclicly permutes these real bosons: $\phi^I_1\to\phi^I_2\to\phi^I_3\to\phi^I_1$. We can define new bosons $\varphi^I\equiv\frac{1}{\sqrt{3}}(\phi^I_1+\phi^I_2+\phi^I_3)$; the other ten real bosons are complexified via linear combinations $\Phi^I\equiv\frac{1}{\sqrt{3}}(\phi^I_1+\omega^2\phi^I_2+\omega\phi^I_3)$ and $(\Phi^I)^\dagger\equiv\frac{1}{\sqrt{3}}(\phi^I_1+\omega\phi^I_2+\omega^2\phi^I_3)$, where $\omega=\exp(2\pi i/3)$. Under $\mathcal{P}_{10}$, φ^I is invariant, while Φ^I $((\Phi^I)^\dagger)$ are eigenstates with eigenvalue ω (ω^2), *i.e.*, $\mathcal{P}_{10}$ acts as a $\mathbf{Z}_3$ twist on Φ^I $((\Phi^I)^\dagger)$.

Next we introduce the following $\mathbf{Z}_3\otimes\mathbf{Z}_2$ twist on the $N1$ model:

$$T_3=(\theta||0|0)(\Theta||\mathcal{P}_2|(-\sqrt{2}/3))(\mathcal{P}_{10}|2/3)\ , \tag{3}$$

$$T_2=(\vec{0}||\sqrt{2}/2|0)(-\mathbf{1}||0^3|\sqrt{2}/2)(0^{15}|0)\ . \tag{4}$$

Here T_3 is a $\mathbf{Z}_3$ twist that acts as follows. The right-moving $\Gamma^{2,2}$ momenta (and the corresponding oscillator excitations) are $\mathbf{Z}_3$ twisted by the twist θ. The left-moving $\Gamma^{2,2}$ momenta are untouched. This is an asymmetric orbifold. The right-moving $\Gamma^{4,4}$ momenta (and the corresponding oscillator excitations) are $\mathbf{Z}_3$ twisted by the twist Θ. The corresponding left-movers are twisted by the outer automorphism $\mathcal{P}_2$, and the last $SU(2)$ real boson η_4 is shifted by $-\sqrt{2}/3$ (this is an order three shift as $\sqrt{2}$ is a root of $SU(2)$). Lastly, the three $SO(10)$s are twisted by their outer automorphism $\mathcal{P}_{10}$, and the $SO(2)$ momenta are shifted by $2/3$ (this is an order three shift as well since 2 is in the identity weight of $SO(2)$). Next, T_2 is a $\mathbf{Z}_2$ twist that acts as follows. The right-moving $\Gamma^{2,2}$ momenta are untouched, whereas their left-moving counterparts are shifted by $\sqrt{2}/2$ (Note that this is a $\mathbf{Z}_2$ shift of the momenta in the $SU(2)$ subgroup of $SU(3)$; the $U(1)$ momenta are separated from those of $SU(2)$ by a single vertical line). $\Gamma^{4,4}$ is asymmetrically twisted: The right-movers are twisted by a diagonal $\mathbf{Z}_2$ twist ($\mathbf{1}$ is a 4×4 identity matrix), whereas the left-movers are shifted ($\sqrt{2}/2$ is a $\mathbf{Z}_2$ shift of the momenta in the last $SU(2)$ subgroup of $SO(8)$; it is separated from the first three $SU(2)$s by a single vertical line).

It is easy to verify that the above T_3 and T_2 twists are compatible with the Wilson lines U_1 and U_2, and that the $N1$ model possesses the corresponding $\mathbf{Z}_3 \otimes \mathbf{Z}_2$ (isomorphic to $\mathbf{Z}_6$) symmetry. The resulting model was described in detail in Refs [6,7], so we will not repeat that discussion here. For illustrative purposes, we reproduce the massless spectrum of this model in the first column of Table I. We shall call this model $T1(1)$, where the the number in the bracket refers to the choice of modulus $h = 1$ in the moduli space that gives the $SO(8)$ in the $N0$ model [7].

The $T1(1)$ model has $N = 1$ space-time supersymmetry, and its gauge group is $SU(2)_1 \otimes SU(2)_3 \otimes SO(10)_3 \otimes U(1)^3$ (the subscripts indicate the levels of the corresponding Kac-Moody algebras). This model is completely anomaly free, and its hidden sector is $SU(2)_1$, whereas the observable sector is $S0(10)_3 \otimes U(1)^2$ (the first and the last $U(1)$s in Table I). $SU(2)_3 \otimes U(1)$ plays the role of the messenger/intermediate sector, or horizontal symmetry. Note that the net number of the chiral $SO(10)_3$ families in this model is $5 - 2 = 3$.

To obtain a three-family $SU(5)_3$ model with a larger hidden sector, we first add the following Wilson line

$$U_3 = T_3' = (\vec{0}||0|\sqrt{\frac{2}{3}})(\mathbf{0'}||(\frac{\sqrt{2}}{3})^3|0)((\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{2}{3})^3|0) \qquad (5)$$

to the $N1$ model. We will refer to the resulting $N = 4$ Narain model as $N6$. In the unshifted sector of the $N6$ model the gauge group is broken from $SU(3) \otimes SO(8) \otimes SO(10)^3 \otimes SO(2)$ down to $SU(3)^2 \otimes SU(5)^3 \otimes U(1)^6$ (Note that $SO(8)$ is broken to $SU(3) \otimes U(1)^2$, and each $SO(10)$ is broken to $SU(5) \otimes U(1)$). However, there are additional gauge bosons that come from the shifted and inverse shifted sectors. These are in the irreps $(\mathbf{3}, \overline{\mathbf{3}})(2)$ and $(\overline{\mathbf{3}}, \mathbf{3})(-2)$ of $SU(3) \otimes SU(3) \otimes U(1)$, where the $U(1)$ charge (which is normalized to $1/\sqrt{6}$) is given in the parentheses. Thus, the gauge symmetry of the $N6$ model is $SU(6) \otimes SU(5)^3 \otimes U(1)^5$. This enhancement of gauge symmetry was made possible by breaking the $SO(10)$ subgroups, so that the resulting $SU(5)$ level-3 models can have enhanced hidden sectors.

Next, we can add the T_3 twist to the $N6$ model. The resulting model has the gauge symmetry $SU(4)_1 \otimes SU(5)_3 \otimes U(1)^3$ (Note that $SU(6)$ is broken down to $SU(4) \otimes U(1)^2$, and four of the $U(1)$s in the $N6$ models have been removed by the T_3 twist). The number of chiral

families of $SU(5)_3$ in this model is 9 as it is the case for other level-3 models constructed from a single $\mathbf{Z}_3$ twist [6,7]. Therefore, we add the T_2 twist to obtain a model with the net number of three families. We will refer to the final model (obtained via orbifolding the $N6$ model by the T_3 and T_2 twists) as $F1(1)$. The $F1(1)$ model has gauge symmetry $SU(3)_1 \otimes SU(5)_3 \otimes U(1)^4$. Its massless spectrum is given in the second column of Table I. Note that in the $F1(1)$ model the $SU(3)$ subgroup arises as a result of the breaking $SU(4) \supset SU(3) \otimes U(1)$. The net number of chiral families of $SU(5)_3$ is three in the $F1(1)$ model. In this table, all the $U(1)$ charges are correlated. For example, by $(\mathbf{1},\mathbf{2},\mathbf{1})(\pm 1, \mp 3, +3, 0)_L$, we mean $(\mathbf{1},\mathbf{2},\mathbf{1})(+1,-3,+3,0)_L$ plus $(\mathbf{1},\mathbf{2},\mathbf{1})(-1,+3,+3,0)_L$.

In working out the spectra of the $F1(1)$ model it is useful to view the Wilson line U_3 as a $\mathbf{Z}_3$ twist T_3' that acts on the $T1(1)$ model, even though this twist consists of shifts only and acts on the lattice freely, *i.e.*, with no fixed points. In this approach we are orbifolding the $N1$ lattice by the $\mathbf{Z}_3 \otimes \mathbf{Z}_2 \otimes \mathbf{Z}_3'$ twist generated by T_3, T_2 and T_3', respectively. Here we find that there are additional possibilities. Indeed, since the orders of T_3 and T_3' are the same (both of them have order three), their respective contributions in the one-loop partition function can have a non-trivial relative phase between them. Let this phase be $\phi(T_3,T_3')$. By this we mean that $3\phi(T_3,T_3') = 0 \pmod 1$ (*i.e*, $\phi(T_3,T_3')$ can be $0, 1/3, 2/3$), and the states that survive the T_3 projection in the T_3' shifted sector must have the T_3 phase $\phi(T_3,T_3')$. Similarly, the states that survive the T_3 projection in the inverse shifted sector $(T_3')^{-1}$ must have the T_3 phase $-\phi(T_3,T_3')$. The string consistency then requires that the states that survive the T_3' projection in the T_3 twisted sector must have the T_3' phase $-\phi(T_3,T_3')$. Similarly, the states that survive the T_3' projection in the inverse twisted sector $(T_3)^{-1}$ must have the T' phase $\phi(T_3,T_3')$. The gauge symmetry of the resulting model depends on the choice of $\phi(T_3,T_3')$. The models with $\phi(T_3,T_3') = 0$ and $\phi(T_3,T_3') = 1/3$ are equivalent. Note that the model with $\phi(T_3,T_3') = 0$ is precisely the $F1(1)$ model. The third choice $\phi(T_3,T_3') = 2/3$ leads to a different model, which we will refer to as $F2(1)$. The $F2(1)$ model has gauge symmetry $SU(2)_1 \otimes SU(2)_1 \otimes SU(5)_3 \otimes U(1)^4$. Its massless spectrum is given in the third column of Table I. Here we point out that another advantage in viewing the U_3 Wilson line as the T_3' twist is that the invariant sublattices and numbers of fixed points for the T_3 and T_2 twists remain the same as in the $T1(1)$ model, so that working out the spectra of the final models becomes easier.

Next we translate the above twists T_3, T_2 and T_3' into the generating vectors V_i and structure constants k_{ij} of the orbifold construction rules derived in Ref [7]. These rules are useful in working out the spectra of the above models when the book-keeping of various phases in the partition function become non-trivial in such asymmetric orbifolds. Thus, the generating vectors are given by

$$V_0 = (-\frac{1}{2}(-\frac{1}{2}\ 0)^3||0_r\ 0_r|0\ 0_r\ 0_r|0^5\ 0_r^5\ 0_r)\ ,$$
$$V_1 = (0(-\frac{1}{3}\ \frac{1}{3})^3||0_r\ 0_r|(\frac{2}{3})\ 0_r\ (-\frac{\sqrt{2}}{3})_r|(\frac{1}{3})^5\ 0_r^5\ (\frac{2}{3})_r)\ ,$$
$$V_2 = (0(0\ 0)(-\frac{1}{2}\ \frac{1}{2})^2||(\frac{\sqrt{2}}{2})_r\ 0_r|0\ 0_r\ (\frac{\sqrt{2}}{2})_r|0^5\ 0_r^5\ 0_r)\ ,$$
$$V_3 = (0(0\ 0)^3||0_r\ (\sqrt{\frac{2}{3}})_r|0\ (\sqrt{\frac{2}{3}})_r\ 0_r|(0^5\ (\frac{1}{\sqrt{3}})_r(\frac{1}{\sqrt{3}})_r(\frac{1}{\sqrt{3}})_r(\frac{1}{\sqrt{3}})_r(\frac{2}{\sqrt{3}})_r\ 0_r)\ ,$$

$$W_1 = (0(0\ \frac{1}{2})^3 || 0_r\ 0_r | (\frac{1}{2})\ 0_r\ 0_r | (\frac{1}{2})^5\ 0_r^5\ 0_r)\ ,$$
$$W_2 = (0(0\ 0)(0\ \frac{1}{2})^2 || 0_r\ 0_r | 0\ 0_r\ 0_r | 0^5\ 0_r^5\ 0_r)\ .$$

Here V_1, V_2 and V_3 correspond to the T_3, T_2 and T_3' twists, respectively. Note that W_3 is a null vector since V_3 consists of shifts only. In the $F1(1)$ and $F2(1)$ models we have chosen $k_{00} = 0$ for definiteness (the alternative choice $k_{00} = 1/2$ would result in equivalent models with the space-time chiralities of the states reversed). To preserve $N = 1$ space-time supersymmetry we must put $k_{20} = 1/2$ (the other choice $k_{20} = 0$ would give models with $N = 0$ space-time supersymmetry). Finally, in $k_{13} = \phi(T_3, T_3')$. Note that $k_{13} = 0$ for the $F1(1)$ model, and $k_{13} = 2/3$ for the $F2(1)$ model. (As we mentioned earlier, the third choice $k_{13} = 1/3$ results in a model which is equivalent to the $F1(1)$ model.) The rest of the structure constants are completely fixed. Here, a remark is in order. In working out the spectra of the $F1(1)$ and $F2(1)$ models, certain care is needed when using the spectrum generating formula of Ref [7]. In particular, the latter is sensitive to the assignment of $U(1)$ charges (in $SO(10) \supset SU(5) \otimes U(1)$) in the untwisted vs twisted sectors. This manifests itself in a slight modification of the spectrum generated formula which is required by string consistency. This ensures that the states in the final models form irreps of the final gauge group. Without such a modification, the states in the final model do *not* form irreps of the final gauge group.

Let us note the following. The $T1(1)$ model that we have started with is only one of the three different $SO(10)_3$ models considered in Ref [7]. If we start from the $SO(10)_3$ model given in the second column of Table I in Ref [7] (which we will refer to as $T2(1)$), and add the T_3' twist, we still get the same $F1(1)$ and $F2(1)$ models given in Table I in this paper. If we start from the third $SO(10)$ model, given in the third column of Table I in Ref [7] (which we will refer to as the $T3$ model), and add the T_3' twist, we get two 3-family $SU(6)_3$ models, which we will refer to as $S1$ and $S2$. The spectra of the $S1$ and $S2$ models are similar to those of the $F1(1)$ and $F2(1)$ models, respectively. The corresponding gauge groups are $SU(3)_1 \otimes SU(6)_3 \otimes U(1)^3$ and $SU(2)_1 \otimes SU(2)_1 \otimes SU(6)_3 \otimes U(1)^3$. One can get these spectra from those of the $F1(1)$ and $F2(1)$ models via replacing $SU(5)_3 \otimes U(1)$ (the last $U(1)$) by $SU(6)_3$. Under the branching $SU(6) \supset SU(5) \otimes U(1)$, $\mathbf{6} = \mathbf{5}(-1) + \mathbf{1}(+5)$ and $\mathbf{15} = \mathbf{5}(+4) + \mathbf{10}(-2)$. Note that the $SU(5)_3 \otimes U(1)$ matter content in the $F1(1)$ and $F2(1)$ models has the underlying $SU(6)$ structure. Similarly, the spectra of the $F1(1)$ and $F2(1)$ models can be derived from the spectra of the $S1$ and $S2$ models by giving the Higgs in the adjoint of $SU(6)_3$ a vacuum expectation value that breaks it to $SU(5)_3 \otimes U(1)$. This should make it clear what the spectra of the $S1$ and $S2$ models are. In particular, the $S1$ model has 6 copies of $\mathbf{15}$, 3 copies of $\overline{\mathbf{15}}$, 9 copies of $\overline{\mathbf{6}}$ and 3 copies of $\mathbf{6}$, while the $S2$ model has 3 copies of $\mathbf{15}$, 12 copies of $\overline{\mathbf{6}}$ and 6 copies of $\mathbf{6}$.

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M	$T1(1)$ $SU(2)^2 \otimes SO(10) \otimes U(1)^3$	$F1(1)$ $SU(3) \otimes SU(5) \otimes U(1)^4$	$F2(1)$ $SU(2)^2 \otimes SU(5) \otimes U(1)^4$
U	$(\mathbf{1},\mathbf{1},\mathbf{45})(0,0,0)$ $(\mathbf{1},\mathbf{3},\mathbf{1})(0,0,0)$ $(\mathbf{1},\mathbf{1},\mathbf{1})(0,-6,0)_L$ $2(\mathbf{1},\mathbf{4},\mathbf{1})(0,+3,0)_L$ $2(\mathbf{1},\mathbf{2},\mathbf{1})(0,-3,0)_L$	$(\mathbf{1},\mathbf{24})(0,0,0,0)$ $2(\mathbf{1},\mathbf{1})(0,0,0,0)_L$ $(\mathbf{1},\mathbf{1})(+6,0,0,0)_L$ $2(\mathbf{1},\mathbf{1})(-3,\pm 3,\pm 3,0)_L$ $2(\overline{\mathbf{3}},\mathbf{1})(+3,-3,+1,0)_L$ $(\mathbf{3},\mathbf{1})(0,0,-4,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{24})(0,0,0,0)$ $2(\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(0,0,-6,0)_L$ $2(\mathbf{1},\mathbf{2},\mathbf{1})(\pm 1,\mp 3,+3,0)_L$ $(\mathbf{2},\mathbf{1},\mathbf{1})(\pm 2,0,+3,0)_L$
$T3$	$2(\mathbf{1},\mathbf{2},\mathbf{16})(0,-1,-1)_L$ $2(\mathbf{1},\mathbf{2},\mathbf{10})(0,-1,+2)_L$ $2(\mathbf{1},\mathbf{2},\mathbf{1})(0,-1,-4)_L$ $(\mathbf{1},\mathbf{1},\mathbf{16})(0,+2,-1)_L$ $(\mathbf{1},\mathbf{1},\mathbf{10})(0,+2,+2)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(0,+2,-4)_L$	$6(\mathbf{1},\mathbf{10})(+1,-1,-1,-2)_L$ $6(\mathbf{1},\mathbf{5})(+1,-1,-1,+4)_L$ $6(\mathbf{1},\mathbf{1})(-2,+1,-1,-5)_L$ $6(\mathbf{1},\overline{\mathbf{5}})(-2,+1,-1,+1)_L$ $3(\mathbf{1},\mathbf{1})(+1,0,+2,-5)_L$ $3(\mathbf{1},\overline{\mathbf{5}})(+1,0,+2,+1)_L$	$3(\mathbf{1},\mathbf{1},\mathbf{10})(0,0,+2,-2)_L$ $3(\mathbf{1},\mathbf{1},\mathbf{5})(0,0,+2,+4)_L$ $6(\mathbf{1},\mathbf{1},\mathbf{1})(\pm 1,\mp 1,-1,-5)_L$ $6(\mathbf{1},\mathbf{1},\overline{\mathbf{5}})(\pm 1,\mp 1,-1,+1)_L$
$T6$	$(\mathbf{1},\mathbf{1},\overline{\mathbf{16}})(\pm 1,+1,+1)_L$ $(\mathbf{1},\mathbf{1},\mathbf{10})(\pm 1,+1,-2)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(\pm 1,+1,+4)_L$	$3(\mathbf{1},\mathbf{1})(+2,+1,+1,+5)_L$ $3(\mathbf{1},\mathbf{5})(+2,+1,+1,-1)_L$ $3(\mathbf{1},\overline{\mathbf{10}})(-1,-1,+1,+2)_L$ $3(\mathbf{1},\overline{\mathbf{5}})(-1,-1,+1,-4)_L$	$3(\mathbf{1},\mathbf{1},\mathbf{1})(\pm 1,\pm 1,+1,+5)_L$ $3(\mathbf{1},\mathbf{1},\mathbf{5})(\pm 1,\pm 1,+1,-1)_L$
$T2$	$(\mathbf{2},\mathbf{2},\mathbf{1})(0,0,0)_L$ $(\mathbf{2},\mathbf{4},\mathbf{1})(0,0,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(\pm 3,-3,0)_L$	$(\mathbf{3},\mathbf{1})(\pm 3,-3,-1,0)_L$ $(\overline{\mathbf{3}},\mathbf{1})(-3,+3,+1,0)_L$ $(\mathbf{1},\mathbf{1})(+3,\pm 3,\mp 3,0)_L$	$(\mathbf{2},\mathbf{2},\mathbf{1})(\pm 1,\mp 3,0,0)_L$ $(\mathbf{1},\mathbf{2},\mathbf{1})(\pm 1,\pm 3,-3,0)_L$
$U(1)$	$(\frac{1}{\sqrt{6}},\ \frac{1}{3\sqrt{2}},\ \frac{1}{6})$	$(\frac{1}{3\sqrt{2}},\ \frac{1}{2\sqrt{3}},\ \frac{1}{2\sqrt{3}},\ \frac{1}{3\sqrt{10}})$	$(\frac{1}{2},\ \frac{1}{2\sqrt{3}},\ \frac{1}{3\sqrt{2}},\ \frac{1}{3\sqrt{10}})$

TABLE I. The massless spectra of the three models: $T1(1)$, $F1(1)$ and $F2(1)$, with gauge symmetries : (1) $SU(2)_1 \otimes SU(2)_3 \otimes SO(10)_3 \otimes U(1)^3$, (2) $SU(3)_1 \otimes SU(5)_3 \otimes U(1)^4$, and (3) $SU(2)_1 \otimes SU(2)_1 \otimes SU(5)_3 \otimes U(1)^4$. Note that double signs (as in $(\mathbf{1},\mathbf{2},\mathbf{1})(\pm 1,\mp 3,-3,0)_L$) are correlated. The $U(1)$ normalization radii are given at the bottom of the table. The graviton, dilaton and gauge supermultiplets are not shown.

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